THEORETICAL GEOMECHANICS

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In memoriam

Professor **Marin DOROBANŢU**

INTRODUCTION

These course notes present some theoretical problems related to the Mechanics of a Continuum Solid Body, of particular importance to Applied Geomechanics, Geological Engineering and Structural Geology. In most cases, only static aspects are discussed, but some dynamic cases are also presented.

As a rule, the modern tensorial approach is used. Some basic elements are reviewed at the beginning of the notes. More details can be found into the excellent books of George Backus and Brian Kennett, also available at Samizdat Press. The linear elasticity and the homogeneity of the continuum solid body are almost thoroughly assumed to be valid, but some elements of Rheology are also presented.

In most cases, the semi-inverse method is used to solve the problems. According to it, the solution is supposed to be of a particular form, as a consequence of the simplified hypothesis previously assumed. It is verified that solution checks both the corresponding equations and the boundary conditions. Based on the Uniqueness Theorem of the Linear Elasticity, it follows the assumed particular solution is just the general solution of the problem. In all the cases discussed here, the assumed simplified hypotheses allow one to obtain simple, analytical solutions. At a first glance, the importance of such solutions is minor with respect to the real cases, where mainly the non-homogeneity of the medium plays a great role. However, the analytical solutions are the basis for deriving finite element algorithms, allowing one to model satisfactory the complex real cases. Such examples are also presented.

The author will kindly appreciate any critical remarks on these notes, being very grateful to Samizdat Press for the possibility to shear his work.

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